C&O 331: Assignment #3 Due date (beginning of class): Feb 29 (Wednesday)

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1. (a) Does there exist a perfect code of length 27 and distance 3 over $GF(27)$? (Explain)
   (b) Does there exist a perfect code of length 28 and distance 3 over $GF(27)$? (Explain)
   (c) Prove that every perfect code must have odd distance.

2. (a) Find the smallest value of n for which there exists a binary linear single error-correcting code of length n that will carry 74 bits of information per codeword.
   (Hint: Consider a parity-check matrix for such a code.)
   (b) Let C be a (not necessarily linear) $[n, M]$-code over an alphabet of size q with distance d. Prove that $M \leq q^{n-d+1}$.
   (Hint: Consider the words you get by deleting the last $d-1$ symbols from each codeword.)

3. Represent the field $GF(4)$ as $\mathbb{Z}_2[x]/(x^2+x+1)$. Let C be the Hamming code of order 2 over $GF(4)$.
   (a) Determine the length n, the dimension k, and the number of codewords M of C.
   (b) Construct a parity check matrix H for C.
   (c) Decode the received vectors $(0, x, 1, x+1, x)$ and $(1, x+1, x, 0, 0)$ to codewords in C using the matrix from part (b).

4. Consider the following parity check matrix H for a binary linear (n, k)-code C with distance d:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

   (a) Determine n, k and d.
   (b) Find a generator matrix G for C.
   (c) Set up a 1-1 correspondence between syndromes and coset leaders. (Use the H given above.)
   (d) Decode each of the following vectors:
      i. $r_1 = (1110110000)$.
      ii. $r_2 = (1011110010)$.

Please note that assignments are not weighted equally. Each problem on each assignment is worth 10 marks. The total marks received on assignments will be added together at the end of the course.

You should make an effort to solve all the problems on your own. You are also welcome to collaborate with your colleagues, and to seek assistance from the teaching assistant or the instructor. However, *all solutions must be written up by yourself*. If you do collaborate, please acknowledge your collaborators in the write-up for each problem. *If you obtain a solution with help from a book, solutions from previous offerings of the course, a web site (including Wikipedia), or elsewhere, please acknowledge your source.*

The assignment is due at the beginning of class on February 29 (Wednesday). Late assignments will not be accepted except in very special circumstances.
