C&O 331: Assignment #4 Due date: March 28 (Wednesday)

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1. Decode each of the following received words using the decoding scheme for C_{24} (the extended binary Golay code).

   (a) $r_1 =$ (0000 0000 0011 1111 1101 1001).

   (b) $r_2 =$ (0011 1000 0000 0100 1100 1110).

   (c) $r_3 =$ (1111 0000 0000 0011 1010 0111).

2. Consider the vector space $V = V_{17}(\mathbb{Z}_2)$.

   (a) Determine the number of cyclic subspaces of V.
   (Recall that there is a one-to-one correspondence between cyclic subspaces of $V_n(F)$ and monic divisors of $x^n - 1$ over F. Hence, if the complete factorization of $x^n - 1$ over F is $x^n - 1 = p_1^{e_1} p_2^{e_2} \cdots p_t^{e_t}$, where the p_i are pairwise distinct monic irreducible polynomials and $e_i \geq 1$, then the number of cyclic subspaces of V is $(e_1 + 1)(e_2 + 1) \cdots (e_t + 1)$.)

   (b) Determine the values of k, $1 \leq k \leq 17$, for which a cyclic subspace of V of dimension k exists.

   (c) Determine the number of these subspaces which have dimension 12.

   (d) Give the generator polynomial for a cyclic subspace of V of dimension 8, if possible.

3. Let C_1 and C_2 be cyclic codes of length n over $F = GF(q)$, with generator polynomials $g_1(x)$ and $g_2(x)$, respectively.

   (a) Prove that $C_1 \bigcap C_2$ is a linear code.

   (b) Prove that $C_1 \bigcap C_2$ is a cyclic code.

   (c) Find the generator polynomial for $C_1 \bigcap C_2$. (Justify your answer.)

4. Let C be a binary $(15, 10)$-cyclic code generated by $g(x) = 1 + x^2 + x^4 + x^5$.

   (a) Show that C is a 2-*cyclic* burst error correcting code.

   (b) Decode the following received vectors using error trapping:

      i. $r_1 =$ (01011 00000 00010).

      ii. $r_2 =$ (10000 10110 10111).

5. (a) Show that $g(x) = 1 + x^2 + x^3 + x^4$ generates a binary cyclic $(7, 3)$-code C which is 2-*cyclic* burst error correcting.

   (b) Let $l(x) = g(x^2) = 1 + x^4 + x^6 + x^8$. Show that $l(x)$ is the generator for a binary cyclic $(14, 6)$-code C^* that is 4-*cyclic* burst error correcting.

   (c) Suppose that C^* is being used for encoding. Let $r =$ (1010000 0011110) be a received vector. Determine the error vector e.

Please note that assignments are not weighted equally. Each problem on each assignment is worth 10 marks. The total marks received on assignments will be added together at the end of the course.

You should make an effort to solve all the problems on your own. You are also welcome to collaborate with your colleagues, and to seek assistance from the teaching assistant or the instructor. However, *all* *solutions must be written up by yourself*. If you do collaborate, please acknowledge your collaborators in the write-up for each problem. *If you obtain a solution with help from a book, solutions from previous offerings of the course, a web site (including Wikipedia), or elsewhere, please acknowledge your source.*

The assignment is due by 5:00pm on March 28. Late assignments will not be accepted except in very special circumstances.
