CO342 ASSIGNMENT #1
DUE: 9:30AM WEDNESDAY 11 MAY 2011

1. Let G be a graph, having at least two edges, with the following properties: (i) for every edge e of G, $G - e$ is connected; and (ii) for any distinct edges e, f of G, $(G - e) - f$ is not connected. Prove that G is a cycle.

2. For every positive integer n with $n \geq 3$, give an example of a graph G on n vertices that has both an avoidable and an unavoidable vertex.

3. Let m and n be positive integers. Determine the avoidable and unavoidable vertices in:

   (a) the complete graph K_n;

   (b) the complete bipartite graph $K_{m,n}$;

   (c) the n-dimensional cube Q_n.

4. Let v be a vertex in the n-dimensional cube Q_n. Determine the avoidable and unavoidable vertices in $Q_n - v$. *Hint: Q_n is a bipartite graph that has a perfect matching, so a maximum matching in $Q_n - v$ has size* $2^{n-1} - 1$.

5. Let T be a tree having at least two vertices. Let v be a vertex of T with degree 1 and let w be its neighbour in T.

   (a) Prove that $\nu(T) = 1 + \nu(T - \{v, w\})$.

   (b) Based on 5a, describe how to find a maximum matching in a tree. (Do not forget to take into account that $T - \{v, w\}$ might not be connected.)

   (c) Based on 5a, or otherwise, prove that a tree has at most one perfect matching.