# CO342 ASSIGNMENT #4
# DUE: IN CLASS WEDNESDAY 1 JUNE 2011

1. Let $\approx$ be the relation on the set $E(G)$ of edges of $G$ defined by: if $e$ and $f$ are in $E(G)$, then $e \approx f$ means either $e = f$ or there is a cycle of $G$ containing both $e$ and $f$.

   Prove that $\approx$ is a transitive relation. (What you need to prove is: if $e$, $f$, and $g$ are edges so that $e \approx f$ and $f \approx g$, then $e \approx g$.)

2. Recall that the *blocks* of a connected graph $G$ were defined on Assignment 3 to be the maximal elements of the subgraphs of $G$ that do not have a cut-vertex. Prove that if $e$ and $f$ are edges in different blocks of $G$, then there is no cycle in $G$ containing both $e$ and $f$.

3. **Bonus Question; not required** Prove that an equivalence class of $\approx$ is precisely the edge set of a block.

4. Let $G$ be a 2-connected graph and let $uv$ be any edge of $G$. Let $G'$ be the graph obtained from $G$ by deleting the edge $uv$ and adding a new vertex $w$ that is adjacent just to $u$ and $v$. (This is called *subdividing the edge* $uv$.) Prove that $G'$ is 2-connected.

   For the last two questions, let $k$ be a positive integer and let $G$ be a graph. Recall that $G$ is $k$-*connected* if:

   (a) $|V(G)| \geq k + 1$; and

   (b) for each subset $W$ of $V(G)$ with $|W| = k - 1$, $G - W$ is connected.

5. Let $k$ be a positive integer and let $G$ be a $k$-connected graph. Let $v_1, v_2, \ldots, v_k$ be distinct vertices of $G$. Create a new graph $H$ from $G$ by adding a new vertex $w$ that is adjacent to precisely $v_1, v_2, \ldots, v_k$. Prove that $H$ is $k$-connected.

6. (a) For each integer $n \geq 3$, give an example of a 2-connected graph $G_n$ so that every cycle in $G_n$ contains all the vertices of $G_n$.

   (b) Prove that if $G$ is a 3-connected graph, then there is a cycle in $G$ that does not contain all the vertices of $G$.