CO342 ASSIGNMENT #5
DUE: IN CLASS WEDNESDAY 8 JUNE 2011

Midterm is Wed 15 June in class.

1. Let G be a k-connected graph and let $u, v_1, v_2, \ldots, v_k$ be $k+1$ distinct vertices in G. Show that there are k paths $P_1, P_2, \ldots, P_k$ in G so that: (i) for $i = 1, 2, \ldots, k$, P_i joins u and v_i; and (ii) for $i \neq j$, P_i and P_j have only u in common. (Hint: Menger's Theorem that we have just proved and Asst. 4 #5.)

2. Let G be a k-connected graph and let A and B be any two sets of vertices, both having size k. Prove that there are k pairwise **totally** disjoint paths $P_1, P_2, \ldots, P_k$ so that each P_i joins a vertex of A to a vertex of B. (Two paths are totally disjoint if they have no vertices in common.)

3. Let G be a graph and k a positive integer. Prove the following.

   (a) Suppose G is $(k+1)$-connected. Prove that, for every vertex v of G, $G - v$ is k-connected.

   (b) Suppose that, for every vertex v of G, $G - v$ is k-connected. Prove that G is $(k+1)$-connected.

4. For each integer $k \geq 2$, find an example of a k-connected graph G_k that has some $k+1$ distinct vertices $v_0, v_1, \ldots, v_k$ that are not all together on a cycle in G_k.

5. Let $k \geq 2$ be an integer, let G be a k-connected graph, and let $v_1, v_2, \ldots, v_k$ be distinct vertices of G. Prove that G has a cycle containing all of $v_1, v_2, \ldots, v_k$. (Hint: use induction on k and Asst. 4 #5. For the base case $k = 2$, you may assume the result from class, which proves this for $k = 2$, so I only care about the inductive step. Be careful: it is common to overlook a case in the induction step.)