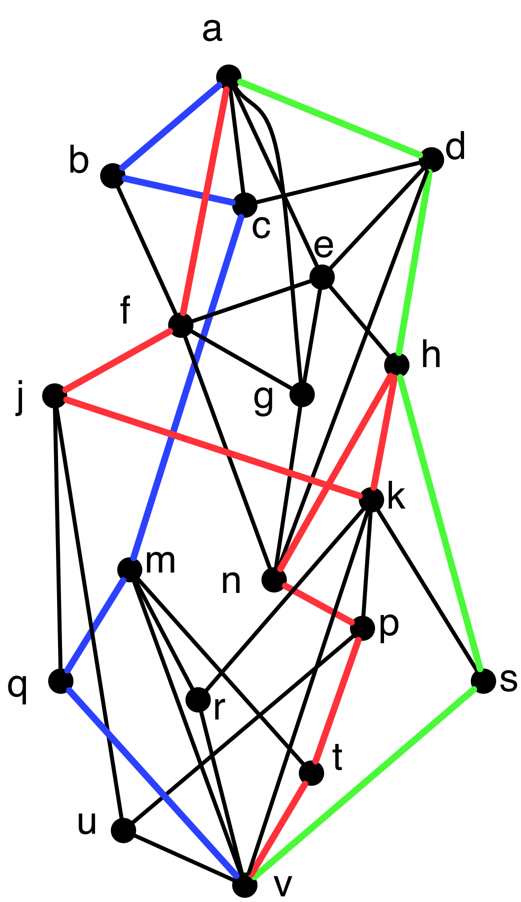
CO342 ASSIGNMENT #6
DUE: IN CLASS WEDNESDAY 22 JUNE 2011

1. In the figure is an example of a graph G. The set of coloured edges represent three pairwise edge-disjoint av-paths in G.



Illustrate the labelling algorithm and use it to produce a largest possible set of pairwise edge-disjoint av-paths in G. (In another file is a copy of the figure, for your convenience.)

2. For each pair of positive integers k and ℓ, with $k > \ell$, give an example of a graph G having a pair of vertices u and v so that:

- a largest set of pairwise edge-disjoint paths in G has size k; and
- there is a vertex w of G, different from u and v, so that the largest set of pairwise edge-disjoint paths in $G - w$ has size ℓ.

3. Let u, v, and w be distinct vertices in a graph G. Suppose there are k pairwise edge-disjoint uv-paths in G and there are k pairwise edge-disjoint vw-paths in G. Prove that there are k pairwise edge-disjoint uw-paths in G.

4. In this exercise, we shall deduce the edge-disjoint version of Menger's Theorem from the internally disjoint version.

   Internally-disjoint Menger's Theorem: For non-adjacent vertices u and v of G, let $\kappa_G(u, v)$ denote the size of a smallest uv-cut. Then there is a set $\mathcal{P}$ of pairwise internally-disjoint uv-paths in G so that $|\mathcal{P}| = \kappa_G(u, v)$.

   Edge disjoint version of Menger's Theorem: let u and v be vertices in a graph G. Then the maximum size of a set of pairwise edge-disjoint uv-paths in G is equal to the minimum size of a set F of edges of G so that $G - F$ has no uv-path.

   Hint. Create a new graph G' as follows:

   - *the vertex set of G' is u, v, and, for each edge e of G, a vertex w_e;*
   - *for each edge e of G incident with u, there is an edge uw_e in G';*
   - *for each edge e of G incident with v, there is an edge vw_e in G';*
   - *if e and e' are edges of G both incident with the vertex x of G, $x \neq u, v$, then w_e and w'_e are adjacent in G'.*

   Show that internally-disjoint paths in G' give edge-disjoint paths in G, and that a set W' of vertices of G' for which u and v are in different components of $G' - W'$ corresponds in G to a set F of edges so that u and v are in different components of $G - F$.