CO342 ASSIGNMENT #8
DUE: IN CLASS WEDNESDAY 6 JULY 2011

1. Let $n \geq 3$ be an integer. Show that every element of $\mathcal{Z}(K_n)$ is a linear combination of edge-sets of 3-cycles in the complete graph K_n with n vertices.

2. Let G be a graph and let H be a 2-connected subgraph of G. Let P be a path in G with length at least one and such that $P \cap H$ consists of just the ends of P. Prove that $H \cup P$ is 2-connected.

3. Let G be a graph and let H be a 2-connected subgraph of G. Let P be a path in G with length at least one and such that $P \cap H$ consists of just the ends of P. Prove that $\dim(\mathcal{Z}(H \cup P)) = 1 + \dim(\mathcal{Z}(H))$.

4. A *cycle double cover* of a graph G is a sequence $(C_1, C_2, \ldots, C_k)$ of cycles in G so that every edge of G appears in exactly two of the C_i. Suppose $(C_1, C_2, \ldots, C_k)$ is a cycle double cover of a graph G.

   (a) Show that $\bigoplus_{i=1}^{k} E(C_i) = \varnothing$.

   (b) Suppose there is a t with $1 < t < k$ so that $\bigoplus_{i=1}^{t} E(C_i) = \varnothing$. Prove:

   - $\bigoplus_{i=t+1}^{k} E(C_i) = \varnothing$; and
   - if C is a cycle with one edge in C_1 and another edge in C_k, show that $E(C)$ is not in the span of $E(C_1), E(C_2), \ldots, E(C_k)$.

5. Let H be a 2-connected subgraph of a graph G and let P be a path in G so that:

   (a) P has both ends in H;

   (b) P is otherwise disjoint from H; and

   (c) $G = H \cup P$.

   Let $(C_1, C_2, \ldots, C_k)$ be a cycle double cover of G so that $\{C_1, C_2, \ldots, C_k\}$ spans the cycle space of G. If P is contained in both C_1 and C_2, but not in any other C_i, prove that $\{C_3, C_4, \ldots, C_k\}$ spans the cycle space of H.