CO342 ASSIGNMENT #9
DUE: IN CLASS WEDNESDAY 13 JULY 2011

Throughout this assignment, we will be dealing with *peripheral cycles*. Let C be a cycle in a graph G.

Definition. 1. A *chord* of C is an edge e of G that is not in C, but both ends of e are in C.

2. C is a *peripheral cycle* if C has no chords and $G - V(C)$ is connected.

1. Determine (with some justification) which cycles in K_n, $K_{m,n}$, and Q_n are chordless and which are peripheral.

2. Let G be a graph embedded in the plane and let C be a peripheral cycle of G. Prove that C bounds a face of G.

3. Let G be a graph in which the length of the shortest cycle is g.

   (a) Prove that if C is a cycle of length at most $2g - 3$, then C has no chord.

   (b) Prove that a cycle C of length at most $2g - 3$ is peripheral if and only if $G - V(C)$ is connected.

4. (a) Let $k \geq 3$ be an integer and let G be a k-connected graph. Prove that if C is a cycle of length less than k, then C is peripheral if and only if C has no chords.

   (b) For each $k \geq 3$, give an example of a k-connected graph that has a cycle of length k that is not peripheral.

   (c) Let $k \geq 3$ be an integer and suppose G is a k-connected graph. Let C be a cycle in G and let v be a vertex of G not in C. Prove there is a set $\mathcal{P}$ of paths from v to distinct vertices of C so that:

   - $\mathcal{P} \geq \min\{k, |V(C)|\}$;
   - any two paths in $\mathcal{P}$ have just v in common; and
   - each path in $\mathcal{P}$ intersects C just at its end in C.