CO442/642 **Assignment 3** **Due: October 19**

Submit your assignment at the start of class. This is a senior level mathematics course, your solutions should be clear, concise, and logically consistent. If your solution is not essentially correct you will get no credit. You may discuss assignment solutions with another student as long as neither of you has yet written a solution; taking written notes during the discussion is considered cheating.

Problem 1:

(a) Let H be a graph with no clique cutset and let H_1 be a graph that is obtained from H by adding an induced ear. Prove that H_1 has no clique cutset.

(b) Prove that, if G is a graph with no clique cutset and G is not complete, then G can be obtained from a hole by adding a sequence of induced ears.

Problem 2: Let G be a simple graph with maximum degree k. Prove that, if there is no cycle whose vertices all have degree k in G, then G is k edge colourable. (Hint: Consider the technical lemma.)

Problem 3: Let G be a cubic graph. Prove that, G is 3 edge colourable if and only if G is the union of two even subgraphs (that is, $E(G) = C_1 \cup C_2$ where $C_1, C_2 \in \mathcal{C}(G)$).

Problem 4: Prove that, if $G = (V, E)$ is a connected plane graph, and T is a spanning tree of G, then $E - E(T)$ is the edge set of a spanning tree in G^*. (Hint: Consider cycles and cuts.)

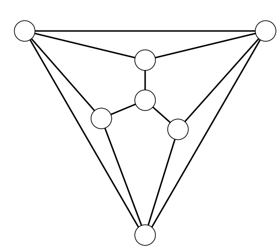
Problem 5: Let G be a plane triangulation.

(a) Prove that, if (G_1, G_2) is a proper separation in G, $G_1 - V(G_2)$ and $G_2 - V(G_1)$ are both connected, and each vertex in $V(G_1) \cap V(G_2)$ has a neighbour in both $V(G_1) - V(G_2)$ and $V(G_2) - V(G_1)$, then $V(G_1) \cap V(G_2)$ induces a cycle in G.

(b) Prove that, if G is simple, then G is 3-connected.

(c) Prove that, if G is simple and has no clique cutset, then G is 4-connected.

Problem 6: [CO642 Only] Show that the following graph is 4-constructable. (Hint: Consider the proof of Hajo's Theorem.)



Problem 7: [CO642 Only] Show that there are infinitely many 4-critical planar graphs of maximum degree 4. (Hint: Consider the graph in Problem 6.)

Problem 8: [Bonus Problem] Prove that, for any graph G, there is a partition (C_1, C_2) of $E(G)$ with $C_1 \in \mathcal{C}$ and $C_2 \in \mathcal{C}^*$. (Hint: This is elementary linear algebra.)