Math 146 Assignment 1
Due 1:00 pm on Wednesday, January 19, 2011

Please submit your assignments in Drop Box #10, Slot#12, outside the tutorial center on the fourth floor of the MC building.

Chapter 1, §1.2

Read the several examples of vector spaces given in the text book. Let F be a field, S be a set. The operations on the vector spaces F^n, $M_{m\times n}(F)$, $\mathcal{F}(S,F)$ and $P(F)$ are as described in the text.

1. Evaluate the following operations.

(i) For $\mathbb{R}^3$: $3(1.1, \sqrt{2}, 3) + (-3)(5.1, \sqrt{2}, 4)$.
(ii) For $M_{2\times 3}(\mathbb{R})$: $3A - 2B$ where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 4 \end{bmatrix}.$$

(iii) For $M_{2\times 3}(\mathbb{Z}_7)$: $3A - 2B$ where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 4 \end{bmatrix}.$$

(iv) For $\mathcal{F}(]0,1[, \mathbb{R})$: $f - g$ where f and g are defined by $f(x) = (x-1)^2$ and $g(x) = (x+1)^2$ for all $x \in]0,1[$.
(v) For $P(\mathbb{C})$: $(5-i)(2x^7 - 6x^4 + 8x^2 - 3x) + (3+i)(x^5 - 2x^3 + 4x + 2)$.

2. Let V and W be vector spaces over a field F. On the Cartesian product $V \times W := \{(v,w) \,|\, v \in V, w \in W\}$ we define the operations

$$(v_1, w_1) + (v_2, w_2) = (v_1 + v_2, w_1 + w_2) \quad \text{and} \quad \lambda(v,w) = (\lambda v, \lambda w).$$

Justify that $V \times W$ is a vector space over F.

3. Let K is a subfield of F. Let V be a vector space over F. Show that V is a vector space over K when we retain the same addition operation and restrict scalar multiplication to scalars from K.

4. (i) Is $\mathbb{Z}_6$ a field? Justify. (ii) Is $\mathbb{Q}$ a subfield of $\mathbb{C}$? (iii) Is $\mathbb{Z}_2$ a subfield of $\mathbb{R}$?

5. Is it true that in a vector space V over F, $x + x = 0$ implies that $x = 0$? Give either a proof or a counter example.

6. Let $P_n(F)$ denote the set of polynomials over F of degree at most n (see page 18). Show that

$$\cup_{n\in\mathbb{N}} P_n(F) = P(F).$$