Math 146 Assignment 2
Due 1:00 pm on Wednesday, January 26, 2011

Please submit your assignments in Drop Box #10, Slot#12, outside the tutorial center on the fourth floor of the MC building.

Chapter 1, §1.3,§1.4,§1.5

1. Justify the following statements

(i) $\{(x, x^2) \,|\, x \in \mathbb{R}\}$ is not a subspace of $\mathbb{R}^2$.
(ii) $\{(x_1, x_2, x_3) \,|\, x_1, x_2, x_3 \in \mathbb{R}, x_1 - 4x_2 > x_3\}$ is not a subspace of $\mathbb{R}^3$.
(iii) The trace of a square matrix A, denoted $tr(A)$, is defined as the sum of all its diagonal entries. The set of all matrices $A \in M_{n\times n}(F)$ satisfying $tr(A) = 0$ is a subspace of $M_{n\times n}(F)$.
(iv) Let S be a non-empty set and F be a field. The set of all $f \in \mathcal{F}(S, F)$ such that $f(s) = 0$ for all but a finite number of elements s of S is a subspace of $\mathcal{F}(S, F)$.

2. Let V be a vector space over F. Let W_1 and W_2 be two subspaces of V. Suppose that $W_1 \cup W_2$ is again a subspace of V. Show that either $W_1 \subset W_2$ or $W_2 \subset W_1$.

3. The textbook on page 26 and 27 contains general instructions in solving the following system of equations.

$$\begin{aligned} a_1 - 2a_2 + 2a_4 - 3a_5 &= 2 \\ 2a_1 - 4a_2 + 2a_3 + 8a_5 &= 6 \\ a_1 - 2a_2 + 3a_3 - 3a_4 + 16a_5 &= 8. \end{aligned}$$

Solve the above system of equations over the field $\mathbb{Z}_5$ according to the given instructions.

4. (i) Consider the vector space $\mathbb{R}$ over the field $\mathbb{R}$. Show that $\sqrt{2} \in span(\{1\})$. (ii) Consider the vector space $\mathbb{R}$ over the field $\mathbb{Q}$, Show that $\sqrt{2} \notin span(\{1\})$.

5. A function f with domain the open interval $]0, \infty[$ and codomain $\mathbb{R}$ is said to be *log-like* if

$$f(st) = f(s) + f(t), \quad \forall s > 0, t > 0.$$

Show that the set of all log-like functions is a subspace of $\mathcal{F}(]0, \infty[, \mathbb{R})$.

6. Let v_1, v_2, v_3 be three distinct vectors of V. Suppose that $\{v_1, v_2\}$, $\{v_2, v_3\}$ and $\{v_1, v_3\}$ are linearly independent sets. Does it imply that $\{v_1, v_2, v_3\}$ is linearly independent? Give a proof or a counter example.