Math 146 Review Exercises (no submission)
Midterm Exam: Monday, February 07, 2011 Time: 7:00-9:00 pm
Location: CPH 3602 for A-M and 3604 for N-Z

note: Two files are included in the Supplementary Notes folder. Please download them to complete your note-taking. The midterm exam will cover materials up to and including those on Friday, Feb 4.

1. (a) Give the five conditions on $\oplus$ for $(V, \oplus, \odot)$ to be a vector space over a field $\mathbb{F}$.

(b) Let $V = \{u \in \mathbb{R} \,|\, u > 0\}$ be the set of all positive real numbers and let $\oplus$ be defined by

$$u \oplus v = uv \text{ for all } u, v \in V,$$

i.e., $\oplus$ is the usual multiplication. Show that V and $\oplus$ satisfy the five conditions.

2. Solve the linear system over the complex field $\mathbb{C}$:

$$\begin{array}{ccccccc} x_1 & - & 2x_2 & + & 2x_3 & = & -1 \\ 2x_1 & + & x_2 & + & x_3 & = & 3 \\ x_1 & + & 0x_2+ & & ix_3 & = & 4. \end{array}$$

3. Let V be the vector space of all functions from $\mathbb{R}$ to $\mathbb{R}$, with the standard operations. Show whether or not $\{f \in V \,|\, f(x+1) = 2f(x) \text{ for all } x\}$ is a subspace of V.

4. Let $\{v_1, v_2, v_3\}$ be a linearly independent subset of a vector space over $\mathbb{C}$. Determine all scalars λ with which $\{v_1 - \lambda v_2, \lambda v_1 + v_2, v_1 + v_2 + v_3\}$ is linearly independent.

5. Determine if the given statement is true or false.

(i) The emptyset is a subspace of $\mathbb{R}^2$.
(ii) There exists a vector space with 6 elements.
(iii) For every subset S of a vector space V, $\text{span}(S) = \text{span}(\text{span}(S))$.
(iv) The union of subspaces is a subspace.
(v) A subset of a linearly independent set is always linearly independent.
(vi) If $S \subset T$, then $span(S) \subset span(T)$.
(vii) $span(S) \subset span(T)$ then $S \subset T$.
(viii) The dimension of the space of all symmetric matrices in $M(4 \times 4, F)$ is 10.
(ix) The dimension of $\mathbb{R}$ over $\mathbb{Q}$ exceeds 4.
(x) The dimension of $\mathcal{F}(\{1, 2, 3, 4\}, F)$ is 4.