Math 146 Assignment 5
Due 1:00 pm on Wednesday, February 16, 2011

Please submit your assignments in Drop Box #10, Slot#12, outside the tutorial center on the fourth floor of the MC building.

1. Read the seven examples on linear maps given in the textbook pp 65-67. (i) Show that $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(a_1, a_2) = (a_1^2, a_2)$ is non-linear. (ii) Parallel to Example 2, write down the formula that gives the rotation of the plane $\mathbb{R}^2$ *clockwise* by θ. (iii) Let $C(\mathbb{R})$ be the vector space of continuous real-valued functions on $\mathbb{R}$ and let $T : C(\mathbb{R}) \to \mathbb{R}$ be defined by $T(f) = \int_{-1}^{1} f(t)dt$ (Example 7). Is the range space $R(T)$ finite dimension? Is the null space (kernel) $N(T)$ finite dimensional?

Definition Let V be a vector space and let $T : V \to V$ be linear. A linear map of a space into itself is called a *linear operator* on V. A subspace W of V is T*-invariant* if $T(w) \in W$ for all $w \in W$ (cf textbook page 77).

2. Let V be a vector space and let be a linear operator on V. Show that the subspaces $\{0\}$, V, $R(T)$, $N(T)$ are all T-invariant.

3. Let T_θ be the rotation of the plane $\mathbb{R}^2$ by an angle θ counterclockwise. Describe all T_θ-invariant subspaces.

4. Let $L : U \to V$ be linear. (i) Suppose that $u_1, u_2, ..., u_n$ are distinct vectors of U. Show that if $v_i := T(u_i)$ are distinct and $\{v_1, ..., v_n\}$ is linearly independent in V, then $\{u_1, ..., u_n\}$ is linearly independent. (ii) Give an example to show that linearly independent $\{u_1, ..., u_n\}$ does not always yield linearly independent $\{v_1, ..., v_n\}$ even if all v_i are distinct.

5. Let V be the vector space of all real sequences $(a_1, a_2, ...)$ (Example 5, section 1.2 of textbook. It is the space $\mathcal{F}(\mathbb{N}, \mathbb{R})$). The *left shift* $T_\ell : V \to V$ is defined by $T_\ell(a_1, a_2, ...) = (a_2, a_3, ...)$ and the *right shift* $T_r : V \to V$ is defined by $T_r(a_1, a_2, ...) = (0, a_1, a_2, ...)$. Are they linear maps? Are they injective? Are they surjective?

6. (i) Give an example of a linear operator L on $M_{2\times 3}(F)$ such that $N(L) = R(L)$. (ii) Show that no linear operator on $\mathbb{R}^3$ can have same null space and range space.

Bonus Question To be submitted **separately** to me in class or to my office. One mark will be accorded to the final grade for those with a full and correct solution. No part marks will be given. Due within **TWO** weeks.

A function $f : \mathbb{R} \to \mathbb{R}$ is called *additive* if $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$. Prove that if f is additive then $f(rx) = rf(x)$ holds for all $r \in \mathbb{Q}$ and $x \in \mathbb{R}$.