Math 146 Assignment 6
Due 1:00 pm on Wednesday, Mar 2, 2011

Please submit your assignments in Drop Box #10, Slot#12, outside the tutorial center on the fourth floor of the MC building.

1. Read about the Lagrange polynomials and Formula on pages 51-53. It begins with the preamble: "Let $c_0, c_1, ..., c_n$ be distinct scalars in an infinite field F. The polynomials

$$f_i(x) = \Pi_{k \neq i} \frac{x - c_k}{c_i - c_k}$$

are called the Lagrange polynomials associated with $c_0, c_1, ..., c_n$. By regarding $f_i(x)$ as a polynomial function $f_i : F \to F$, we see that ...."

Consider the vector space $\mathcal{P}(F)$ of all polynomials over F and the vector space $\mathcal{F}(F, F)$ of all functions from F to F. Let $\phi : \mathcal{P}(F) \to \mathcal{F}(F, F)$ be defined by $\phi(p(x))$ = the function obtained when $p(x)$ is regarded as a polynomial function from F to F.

(i) Is ϕ a linear map? (sufficient to say yes or no).

(ii) Is ϕ injective and would the answer dependent upon whether F is an infinite field or not?

2. (i) Over the real field, use the Lagrange interpolation formula to construct the polynomial of smallest degree whose graph contains the following points:
$(-4, 24), (1, 9), (3, 3)$. [Textbook §1.6#10]

(ii) Does the above question make sense over every field?

3. In the first question of §2.1 of the textbook on page 74, for each false statement, give a counter-example.

4. (i) Let V be an n-dimensional vector space and let T be a linear operator on V. Suppose that W is a T-invariant subspace of V having dimension k. Show that there is a basis β for V such that $[T]_\beta^\beta$ has the form

$$\begin{bmatrix} A & B \\ O & C \end{bmatrix}.$$

where A is a $k \times k$ matrix and O is the $(n-k) \times k$ zero matrix [Textbook §2.2 #11].

(ii) Could you offer a statement about the converse?

5. Textbook §2.3 #1 on page 96 and provide a counter example when a statement is false.

6. Textbook §2.3 #13 on page 97. Prove that the trace function satisfies $tr(AB) = tr(BA)$.

Bonus Question To be submitted **separately** to me in class or to my office. One mark will be accorded to the final grade for those with a full and correct solution. No part marks will be given. Due March 4, 2011.

The following properties may be assumed about the real numbers: between any two distinct real numbers there are rational numbers and there are irrational numbers. You may quote basic calculus facts and results.

Recall that a function $f : \mathbb{R} \to \mathbb{R}$ is *additive* if $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$. You may refer to the fact that an additive function f satisfies $f(rx) = rf(x)$ for all rational r and real x.

Let f be additive. (i) Show that if f is continuous at some x_0, then it is continuous at every x. (ii) Show that if f is continuous at every x, then it has the form

$$f(x) = cx$$

for all x. Here c is a constant. (iii) Show that there exists an additive function which is discontinuous at every x.