Math 146 Assignment 8
Due 1:00 pm on Wednesday, Mar 16, 2011

Please submit your assignments in Drop Box #10, Slot#12, outside the tutorial center on the fourth floor of the MC building.

1. The matrix

$$A = \begin{bmatrix} 2 & 8 & 1 & 0 & 7 & 0 \\ -3 & -12 & 0 & 2 & 2 & 0 \\ 5 & 20 & -2 & -1 & 0 & 0 \end{bmatrix}$$

has reduced row echelon form

$$B = \begin{bmatrix} 1 & 4 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 & 4 & 0 \end{bmatrix}.$$

(i) Find the rank and nullity of A. (ii) Let A_i $(i = 1, ..., 6)$ denote the six column vectors of A and let W be the span of $\{A_1, ..., A_6\}$. List two subsets of $\{A_1, ..., A_6\}$ which are bases for W. (iii) How many subsets of $\{A_1, ..., A_6\}$ are bases for W? (Just give the final count).

2. Let

$$A = \begin{bmatrix} 2 & 8 & 1 \\ -3 & -12 & 0 \\ 5 & 20 & -2 \end{bmatrix}.$$

Consider the linear operator L_A on $\mathbb{R}^3$ which is the left multiplication by A.
(i) Find the rank and nullity of L_A. (ii) Is L_A an isomorphism?

3. The sequence of row operations $4R_1 \to R_1$, $R_2 <-> R_3$, $(-2)R_2 + R_1 \to R_1$, performed in this order, brings a matrix A to

$$B = \begin{bmatrix} 1 & 5 & 6 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Find A.

4. (i) Q is an $n \times n$ matrix of rank n. Determine its RREF (reduced row echelon form). (ii) Let Q be an invertible $n \times n$ matrix and A be an $n \times m$ matrix. Prove that QA and A have the same RREF. (iii) Prove or disprove the statement: If P is an invertible $m \times m$ matrix and A is an $n \times m$ matrix, then AP and A have the same RREF.

5. Express

$$A = \begin{bmatrix} i & 2 \\ 3 & 4 \end{bmatrix}$$

as a product of elementary matrices and compute the inverse of A.