Math 147 Assignment 10 - **Due Friday Nov. 26, 2010**

1. Show that $1 - \sin x \le \exp x$ for all $x \ge 0$.

2. Find the n'th degree Taylor polynomial for the function $\ln(1-x)$ at $a = 0$.

3. Suppose $c \neq 0$. Let

$$f(x) = \left(\frac{1-cx}{1+cx}\right)^{1/x} \quad \text{for } x \in (-1/c, 1/c),\ x \neq 0.$$

   Is there a way to define $f(0)$ so that f is continuous at 0? Explain. What would be the value of $f(0)$?

4. (a) We say that the sequence (z_n) tends to ∞ if for every $M > 0$ there is some positive integer N such that $z_n > M$ for all $n \ge N$.

   Prove that $\lim_{x\to a} F(x) = \infty$ if and only if whenever the sequence $(x_n) \to a$, then the sequence $(F(x_n))$ tends to ∞.

   (b) Suppose there is some $\delta > 0$ such that f and g are differentiable on $[a-\delta, a+\delta]$, except possibly at a, and $g(x)$, $g'(x)$ are non-zero on $[a-\delta, a+\delta]$, except possibly at a. Assume that $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$. Prove that

$$\text{if } \lim_{x\to a} \frac{f'(x)}{g'(x)} = \infty, \text{ then } \lim_{x\to a} \frac{f(x)}{g(x)} = \infty.$$

5. (a) Suppose $h'(c) = 0$, $h''(c)$ exists and h has a local minimum at c. Show that $h''(c) \ge 0$.

   (b) Suppose that f is a twice differentiable function satisfying

$$f''(x) + f'(x)g(x) - f(x) = 0$$

   for some function g. Prove that if f is 0 at two points, then f is 0 on the interval between them.

6. Bonus: Suppose f is differentiable, $\lim_{n\to\infty} f(n) = L$ ($\in \mathbb{R}$) and $\lim_{x\to\infty} f'(x) = K$ ($\in \mathbb{R}$). Prove that $K = 0$.