Math 147 Assignment 11 - Due Friday Dec. 3, 2010

1. Assume that $f(x)$ is continuous on $[3,5]$, $f(3) = 2$ and $f'(x) = 1/(1+x^3)$ on $(3,5)$. Show that
$$\frac{127}{63} \leq f(5) \leq \frac{29}{14}.$$

2. (a) Show that $\cos 1$ is irrational.

   (b) Estimate $\cos 1$ to within 10^{-12}. Leave your answer as a sum.

3. Show that
$$\lim_{x\to 0} \frac{x^2 \sin(x^3) - x^5}{x^{11}} = \frac{1}{3!}.$$

   (Suggestion: Use Taylor's theorem rather than L'Hopital's rule.)

4. Show that Newton's method will fail with the function $y = x^{1/3}$ if any $x_0 \neq 0$ is chosen.

5. Suppose f' is strictly increasing. Show that each tangent line to f intersects f only once.

6. Suppose $f(x) - f''(x) = 0$ for all x and $f(0) = f'(0) = 0$. Using Taylor polynomials, show that $f(x) = 0$ for all x.

7. Bonus. Prove that if $|f|$ is differentiable at a and f is continuous at a, then f is also differentiable at a. Hint: It suffices to consider only a with $f(a) = 0$. Why?