Math 147 Assignment 5 - **Not to be handed in**

1. (a) Show that
$$\max(f(x), g(x)) = \frac{(f+g)(x) + |(f-g)(x)|}{2}.$$
(b) Prove that if $f, g : \mathbb{R} \to \mathbb{R}$ are continuous functions, then the function $\max(f(x), g(x)) : \mathbb{R} \to \mathbb{R}$ is also continuous.

2. Suppose that
$$\sin(\pi x) \leq f(x) \leq \frac{1}{4x(1-x)}$$
for all $x \in (0,1)$. Determine $f(1/2)$ and $\lim_{x\to 1/2} f(x)$. Is f continuous at $x = 1/2$? (You can assume $\sin x$ is a continuous function.)

3. Show that the polynomial $p(x) = x^4 + 7x^3 - 9$ has at least two real roots.

4. Assume that f is continuous on $[0,1]$ and that $0 \leq f(x) \leq 1$ for all $x \in [0,1]$. Prove there exists a $c \in [0,1]$ such that $f(c) = c$.

5. Suppose f and g are continuous on $\mathbb{R}$ and let $S = \{x \in \mathbb{R} : f(x) = g(x)\}$.Supppose $x_n \in S$ for all $n \in \mathbb{N}$ and the sequence $(x_n)_{n=1}^{\infty}$ converges to x_0. Show that $x_0 \in S$.

6. Let $h : [a,b] \to \mathbb{R}$ be a continuous function and assume that for every $x \in [a,b]$ there exists $y \in [a,b]$ such that $|h(y)| \leq \frac{1}{2}|h(x)|$. Prove there is some $c \in [a,b]$ with $h(c) = 0$.