Math 147 Assignment 7 - Due Friday November 5, 2010

1. Differentiate the following functions

   (a) $\ln(\tan x)$

   (b) $\sin(2x)/\sqrt{x^2+1}$

2. Suppose g is differentiable at a and $g(a) \neq 0$. Use the definition to prove

$$\left(\frac{1}{g}\right)'(a) = \frac{-g'(a)}{(g(a))^2}.$$

3. Let

$$f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}.$$

   Prove f is differentiable at 0.

4. Let

$$g(x) = \begin{cases} x^2 \sin(1/x^2) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}.$$

   Show that g is differentiable everywhere, but that its derivative is not continuous at $x = 0$.

5. (a) Suppose $\alpha > 1$ and that $|f(x)| \leq |x|^\alpha$ for all $x \in \mathbb{R}$. Show that f is differentiable.

   (b) Let $0 < \beta < 1$. Prove that if g satisfies $|g(x)| \geq |x|^\beta$ for all $x \in \mathbb{R}$ and $g(0) = 0$, then g is not differentiable at 0.

6. Suppose $n \in \mathbb{N}$. Assume $f(x) = x^n$ for $x \geq 0$ and $f(x) = 0$ for $x \leq 0$. Prove that $f^{(n-1)}$ exists and find a formula for it, but that $f^{(n)}(0)$ does not exist.