Math 147 Assignment 8 - Due Friday Nov. 12, 2010

1. Prove that the set $\{0, 1, 1/2, 1/3, 1/4, ...\}$ is closed, but that the set $\{1, 1/2, 1/3, 1/4, ...\}$ is neither open nor closed. (See assignment 6 for definitions.)

2. Differentiate:

   (a) $y = \arctan(e^{2x})$

   (b) $y = x^{\ln x}$ for $x > 0$

   (c) $y = 2^{\arcsin x}$

3. A function f is called *even* if for all x, $f(x) = f(-x)$ and called *odd* if $f(x) = -f(-x)$. Show that if f is differentiable and even, then f' is odd.

4. Find all the local minimum and maximum points for the function

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational or } x = 0 \\ 1/q & \text{if } x = p/q \text{ in lowest terms, } q \in \mathbb{N} \end{cases}.$$

5. Sue must cross a circular lake of radius one kilometre. She can row across at 2 km per hour or walk around at 4 km per hour, or she can row part way and walk the rest. What route should she take to cross as quickly as possible? (see diagram below)

6. Let $f(x) = (x-1)^{2/3} + (x+1)^{2/3}$. Find the intervals of increase and decrease, critical and singular points, and the local extrema. Graph the function.

7. Suppose f is differentiable on $[a, b]$.

   (a) Show that if f has a local minimum at a, then $f'(a) \geq 0$, while if f has a local minimum at b, then $f'(b) \leq 0$.

   (b) Suppose that $f'(a) < 0 < f'(b)$. Show that there must be some $c \in (a, b)$ with $f'(c) = 0$. (Hint: Apply the EVT to f. Do you see why you can't apply IVT to f'?)

   (c) Suppose $f'(a) < z < f'(b)$. Show there is some $c \in (a, b)$ with $f'(c) = z$. (Hint: Apply (b) with a suitable function.)

   Comment: This result is known as Darboux's theorem and shows that derivatives have the same intermediate value property that we proved for continuous functions.

8. Bonus: Suppose $f'(x) \geq M > 0$ for all $x \in [0, 1]$. Show there is an interval of length $1/4$ on which $|f| \geq M/4$.

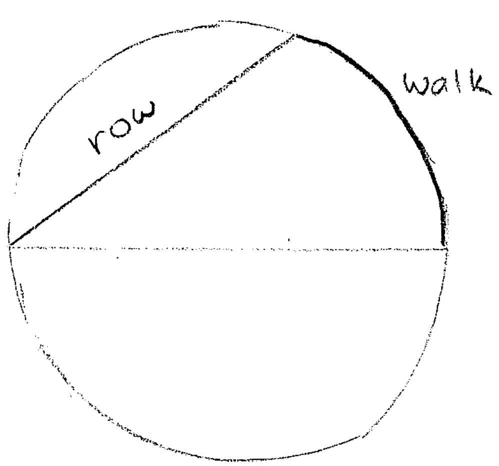

Diagram for ques 5