

# Math 147 Assignment 9 - Due Friday Nov. 19, 2010

1. Find the following limits:

(a)  $\lim_{x \rightarrow 0} \frac{e^x - 2^x}{\arctan x}$

(b)  $\lim_{x \rightarrow \infty} x e^{1/x} - x$

(c)  $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x}\right)^{1/x^2}$

2. Suppose  $g(x) = e^x + 3x$ .

(a) Show that  $g$  is invertible.

(b) Find the domain and range of  $g^{-1}$ . What is  $g^{-1}(e + 3)$ ?

(c) Find  $(g^{-1})'(e + 3)$

3. Let  $a > 0$ . Show that the maximum value of

$$f(x) = \frac{1}{1 + |x|} + \frac{1}{1 + |x - a|}$$

is  $(2 + a)/(1 + a)$ .

4. Find the local and global extrema and inflection points of  $h(x) = x(\log |x|)^2$ . Graph  $h$ .

5. Suppose

$$f(x) = \begin{cases} x^4 \sin^2 1/x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} .$$

(a) Note that 0 is the global minimum value for  $f$ .

(b) Show that  $f'(0) = f''(0) = 0$ .

(c) Prove that there is no  $\delta > 0$  such that  $f$  is increasing on  $[0, \delta]$ .

6. Suppose  $g(0) = 0$  and  $g'$  is a strictly increasing function. Prove that  $f(x) = g(x)/x$  is strictly increasing on  $(0, \infty)$ . (Hint: Look at  $f'$  and apply MVT to  $g(x) - g(0)$ .)

7. Let

$$F(x) = \begin{cases} \exp(-1/x^2) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} .$$

(a) Prove  $F'(0) = 0$ .

(b) Bonus: Prove  $F^{(n)}(0) = 0$  for all  $n \in \mathbb{N}$ .