MATH 148 **Assignment 3** Due: Friday, February 4

Reminder.
We have a $1\frac{1}{2}$ hour mid-term coming up on

Tuesday, February 14 from 7 to 8:30 p.m.

It will count for 20% of your final grade.

This assignment is supposed to help you learn how to integrate.

1. Find the following indefinite integrals.

   (a) $\displaystyle\int \frac{\ln x}{x}\,dx$ (b) $\displaystyle\int \tan x\,dx$ (c) $\displaystyle\int \frac{dx}{1+\sin x}$ (d) $\displaystyle\int \frac{4x^2+3x+2}{x+1}\,dx$

   (e) $\displaystyle\int \frac{dx}{1+e^x}$ (f) $\displaystyle\int x^3 e^{x^2}\,dx$ (g) $\displaystyle\int \arctan x\,dx$ (h) $\displaystyle\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx$

2. Do these indefinite integrals by parts.

   (a) $\displaystyle\int e^x \sin x\,dx$ (b) $\displaystyle\int x \arctan x\,dx$ (c) $\displaystyle\int x(\ln x)^2\,dx$

3. Find these indefinite integrals by following the hints.

   (a) $\displaystyle\int \frac{dx}{\cos x}$ Multiply top and bottom by $\cos x$, then do a bit of trig.

   (b) $\displaystyle\int \sec^3 x\,dx$ Try parts keeping in mind that $\sec^2 x\,dx = d\tan x$

   (c) $\displaystyle\int \frac{dx}{\sqrt{x^2-1}}$ Put $x = 1/\cos\theta = \sec\theta$, then trig.

   (d) $\displaystyle\int \frac{d\theta}{2+\sin\theta}$ Try $\theta = 2\arctan x$.

   (e) $\displaystyle\int \frac{1+e^x}{1-e^x}\,dx$ Try $u = \sqrt{1+e^x}$ and see what pops out.

   (f) $\displaystyle\int \sqrt{x^2+1}\,dx$ Try $x = \tan u$.

4. Find these Riemann integrals.

   (a) $\displaystyle\int_1^2 x\sqrt{x-1}\,dx$ (b) $\displaystyle\int_0^{\pi^2} \sin(\sqrt{x})\,dx$ (c) $\displaystyle\int_0^3 \frac{dx}{\sqrt{9+x^2}}$ (d) $\displaystyle\int_1^e (\ln x)^2\,dx$

5. Integrate the following rational functions using the suggestions provided.

   (a) $\displaystyle\int \frac{dx}{x^2+2x+3}$
   Complete the square of the bottom and make a little substitution to create an $\arctan$ situation.

   (b) $\displaystyle\int \frac{dx}{x(x^2+1)}$
   Solve $\displaystyle\frac{1}{x(x^2+1)} = \frac{a}{x} + \frac{b}{x^2+1} + \frac{cx}{x^2+1}$ for a, b, c.
   Then do one piece at a time.

   (c) $\displaystyle\int \frac{dx}{(x+1)^2(x+2)}$
   Solve $\displaystyle\frac{1}{(x+1)^2(x+2)} = \frac{a}{x+1} + \frac{b}{(x+1)^2} + \frac{c}{x+2}$ for a, b, c.
   Then do one piece at a time.

6. Here's a couple more...

   (a) $\displaystyle\int (\arcsin x)^2\,dx$ (b) $\displaystyle\int \ln(x+\sqrt{x})\,dx$

Remark.
There are numerous examples of integrals to try in any calculus book, and I encourage you to practice quite a few. With that in mind, don't forget there are always functions whose indefinite integrals can never be found through tricks, such as substitutions, parts, or reductions. In fact there are no elementary formulas available at all. Such beasts include

$$\int \frac{\sin x}{x}\,dx, \int \sqrt{1+x^3}\,dx, \int e^{x^2}\,dx \text{ and } \int \sin x^2\,dx$$

So don't feel dumb if there happen to be some you can't do.

BONUSES

Bonus problems are not required to be handed in. They are meant to stimulate your interest. Bonus problems should not be handed in with the regular assignment. Hand them in separately to me as soon as you get them done. For bonus problems, you should try to do them without getting help. Also be sure to get the regular assignments done well before you worry about the bonus problems.

Hand these in by the end of February.

4. Let $f(x) = \cos(1/x)$ for $x \neq 0$ and $f(0) = 0$. Is the integral function $g(x) = \int_0^x f$ differentiable at 0?

5. Does the improper integral $\int_1^\infty \left| \frac{\sin t}{\sqrt{t}} \right| \, dt$ converge?