1: Let $p_1(x) = x-1$, $p_2(x) = \frac{1}{2}(x^2-3x)$ and $p_3(x) = \frac{1}{2}(x^3-3x^2+2)$. Find the polynomial $f \in \mathrm{Span}\{p_1,p_2,p_3\}$ which minimizes the sum $\sum_{i=1}^{5}\big(f(a_i)-b_i\big)^2$ for the 5 points (a_i,b_i) given below

i	1	2	3	4	5
a_i	-1	0	1	2	3
b_i	-2	1	2	0	1

2: (a) Find the perimeter of the regular hexagon on $\mathbf{S}^2$ with interior angles equal to $\frac{5\pi}{6}$.
(b) Find the area of the regular hexagon on $\mathbf{S}^2$ with sides of length $\frac{\pi}{6}$.

3: (a) Let $u = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix}$, $v = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix}$ and $w = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix}$. Find the area of the triangle T on $\mathbf{S}^2$ given by

$$T = \left\{x \in \mathbf{S}^2 \,\middle|\, \mathrm{dist}(x,u) \le \tfrac{\pi}{2},\ \mathrm{dist}(x,v) \le \tfrac{\pi}{2} \text{ and } \mathrm{dist}(x,w) \le \tfrac{\pi}{2}\right\}.$$

(b) Let $u = \frac{1}{\sqrt{6}}\begin{pmatrix}2\\1\\-1\end{pmatrix}$, $v = \frac{1}{\sqrt{6}}\begin{pmatrix}1\\2\\1\end{pmatrix}$ and $w = \frac{1}{\sqrt{6}}\begin{pmatrix}1\\-1\\2\end{pmatrix}$. Find the circumcenter of triangle $[u,v,w]$ on $\mathbf{S}^2$.

4: (a) Let R be the radius of the Earth, in meters ($R \cong 6{,}370{,}000$). We describe the position of a point on the Earth in terms of its longitude θ (with $\theta = 0$ at Greenwitch, England and $\theta = \frac{\pi}{2}$ somewhere in Bangladesh) and its latitude ϕ (with $\phi = 0$ at the equator and $\phi = \frac{\pi}{2}$ at the north pole). Find the distance (expressed as a multiple of R) and the bearing (expressed as an angle north of east) from the point at $(\theta,\phi) = (\frac{\pi}{3},\frac{\pi}{6})$ to the point at $(\theta,\phi) = (\frac{\pi}{2},\frac{\pi}{4})$.
(b) Find the radius R of a sphere on which there is a regular (equilateral) triangle with sides of length 1 and angles equal to $\frac{2\pi}{5}$.

5: Let $u_1,u_2,\cdots,u_{n-2} \in \mathbf{R}^n$ and let $A = \big(u_1,u_2,\cdots,u_{n-2}\big) \in M_{n\times(n-2)}(\mathbf{R})$. For $i<j$, let $A^{i,j}$ denote the $(n-2)\times(n-2)$ matrix obtained from A by removing the i^{th} and j^{th} rows. Note that $\{u_1,\cdots,u_{n-2}\}$ is linearly independent if and only if $A^{i,j}$ is invertible for some $i<j$. Find a formula for an $n\times n$ matrix B with the property that if $\{u_1,\cdots,u_{n-2}\}$ is linearly dependent then $B=0$ and if $\{u_1,\cdots,u_{n-2}\}$ is linearly independent then for all $i<j$, if $A^{i,j}$ is invertible the the i^{th} and j^{th} columns of B form a basis for $(\mathrm{Col}A)^\perp$.