Math 249
Assignment 4

Due: Wednesday, February 9

1. In the string game from the lectures and notes, determine the probability that the string abb occurs before bba and also determine the probability that bbb occurs before bba.

2. Determine the generating series for the 01-strings that do not contain 01001 or 1101 as a substring. From this, determine a linear recurrence for the number of such strings.

3. Let $(f_n)_{n\geq 0}$ and $(g_n)_{n\geq 0}$ be sequences with generating series $F(x)$ and $G(x)$ respectively and suppose that there are constants $a_1, \ldots, a_k$ such that

$$f_n + a_1 f_{n-1} + \cdots + a_k f_{n-k} = g_n, \quad n \geq k.$$

   Prove that there are polynomials $a(x)$ and $b(x)$ such that $a(x)$ is invertible as a power series and

$$a(x)F(x) = G(x) + b(x).$$

4. Use the technique from the previous problem to find the generating function for the series (f_n) such that $f_0 = f_1 = 1$ and

$$f_n - f_{n-1} - f_{n-2} = 2^n, \quad n \geq 2.$$

5. Use partial fractions to derive an explicit formula for the terms of the series in the previous question.