Math 249
Assignment 7

Due: Wednesday, March 9

1. Prove that the generating function for self-conjugate partitions with largest part at most k is

$$\sum_{j=0}^{k} q^{j^2} \begin{bmatrix} k \\ j \end{bmatrix}_{q^2} = (1+q)(1+q^3)\cdots(1+q^{2k-1}).$$

   [There are two things to be proved here.]

2. Establish a bijection between partitions of n with all parts odd and partitions of n with distinct parts based on the following example:

$$7+7+7+5+5+5+5+5+3+3+3+3+3+3+1+1$$
$$= (2+1)7 + (4+1)5 + (4+2)3 + (2)1$$
$$= 14+7+20+5+12+6+2.$$

   (In the second step we are essentially writing the multiplicity of each part in binary.)

3. The *diameter* of a graph is the maximum distance between two vertices. Show that if G has diameter at least four, then the diameter of $\overline{G}$ is at most two. Give an example where G and $\overline{G}$ both have diameter three.

4. Let V be the vector space of dimension d over $\mathbb{Z}_2$ and asume $S \subseteq V \backslash 0$. Define $G(S)$ to be the graph with vertex set V, where two vectors u and v are adjacent if and only if $u + v \in S$. [We say that $G(S)$ is a *cubelike graph*.] Prove that $G(S)$ is connected if and only if S spans V.