Math 249
Assignment 8

Due: Wednesday, March 16

1. Consider a walk on the graph X which starts at u and continues for as long as it can without using the same edge twice. If u has odd degree, show that the vertex on which this walk finishes must have odd degree, and is not u. Using this, prove that any graph has an even number of vertices of odd degree.

2. Suppose $G(S)$ is cubelike and $a \in V$. Show that the map $\tau_a : V \to V$ given by

$$\tau_a(u) = u + a$$

   is an automorphism of $G(S)$.

3. Let m be a positive integer and assume $n = 3m+2$. Let S and T be the subsets of $\mathbb{Z}_n$ defined by

$$S := \{i \in \mathbb{Z}_n : i \equiv 1 \mod 3\}.$$

   and let T consist of elements of $\mathbb{Z}_n$ between $m+1$ and $2m+1$ (inclusive). Prove that the circulant with connection set S is isomorphic to the circulant with connection set T.

4. Let T be a tree and let $\mathscr{P}$ be a set of paths in T. If each pair of paths in $\mathscr{P}$ has a common vertex, prove that there is a vertex of T that lies on all the paths in $\mathscr{P}$.

5. Let $P(G,k)$ denote the number of k-colourings of the graph G. Prove that if $e \in E(G)$, then

$$P(G,k) = P(G \setminus e, k) - P(G/e, k).$$

   Deduce that $P(G,k)$ is a polynomial in k, and determine this polynomial for the path of length n.