Math 249
Assignment 9

Due: Wednesday, March 23

1. (a) Prove that a planar graph with girth at least six must have a vertex of degree at most two.

   (b) Using the previous part, show that a planar graph with girth at least six is 3-colorable.

2. Show that a cubic planar graph with girth at least five has at least 12 faces of degree 5, and that equality holds if there are no faces of degree greater than 6.

3. Let n_i denote the number of vertices of degree i in the graph G. If G has n vertices and e edges, prove that
$$\sum_{i \geq 3} (i-2) n_i = 2e - 2n + n_1 + 2n_0.$$
Deduce that every tree has at least two vertices with degree 1, and that equality holds if and only if it is a path.

4. Show that in any graph with at least two vertices, there are two vertices of the same degree.

5. A planar map is *self-dual* if it is isomorphic to its dual. Show that if G has a self-dual embedding in the plane, it is not bipartite.