PMath 346 ASSIGNMENT 1 Due Jan 20

1. Let G be an abelian group of order pq, where p and q are distinct primes. Prove that G is cyclic.

2. Let G be a group in which every element has order 1 or 2. Prove that G is abelian.

3. Prove that if G is a finite group of even order, then G has an element of order 2.

4. Let $\mathbb{Q}_2 = \{ \frac{a}{b} \in \mathbb{Q} : a, b \in \mathbb{Z}$ and b is a power of 2.

   a) Prove that $\mathbb{Q}_2$ is not cyclic

   b) Prove that if H is a proper subgroup of $\mathbb{Q}_2$ which contains $\mathbb{Z}$, then H is cyclic

5. Let G be a group with a nonempty subset H. Define a relation $\sim$ on G by $a \sim b$ if $ab^{-1} \in H$. Show that if $\sim$ is an equivalence relation, then H is a subgroup of G

6. Find all normal subgroups of $S_3 \times S_3$