

# PMath 346 Assignment 4 - Due Feb 29

1. Describe the Sylow  $p$ -subgroups of  $A_5$ ,  $p=2,3,5$ . How many of each are there?

2. What is the smallest integer  $n$  such that  $D_5$  (the dihedral group of order 10) is isomorphic to a subgroup of  $S_n$ ? Why?

3. Let  $G$  be the group of matrices  $\left\{ \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}; a, b, c \in \mathbb{Z}_2 \right\}$  with matrix

multiplication

i) Prove that  $Z(G) = \left\{ \begin{pmatrix} 1 & 0 & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; b \in \mathbb{Z}_2 \right\}$

ii) Prove that  $G$  is isomorphic to  $D_4$ .

4. Prove that the following 8 matrices (in  $M_4(\mathbb{R})$ ) form a group under matrix multiplication:

$$\left\{ \pm \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \pm \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \pm \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \pm \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \right\}$$

Prove that this group is not isomorphic to  $D_4$ .

5. Prove that a group of order 66 must have an element of order 33.