PMath 346   Assignment 5   Due Mar 14

1. Prove that the Sylow 2-subgroups of S_4 are isomorphic to D_4.

2. A molecule is in the shape of a regular 15-gon with an atom at each vertex. If there are t elements to choose from, how many possible molecules are there?

3. Let G be a finite group for which the only automorphism is the identity map. Prove that $G = \{1\}$ or $G \cong C_2$.

4. Describe up to isomorphism all abelian groups of order 144.

5. In the group $C_4 \times C_9 \times C_3 \times C_3$
    a) how many elements are there of order 9?
    b) how many elements are there of order 6?

6. Prove that in the Quaternion group, the intersection of two nontrivial subgroups is nontrivial (ie $\neq \{1\}$).