

PMath 346 Assignment 6 Due Mar 28

1. Use the semidirect product construction to explicitly construct a nonabelian group of order 21. Each element should have the form  $a^i b^j$   $i \in \{0, \dots, 6\}$ ,  $j \in \{0, 1, 2\}$ . Give a formula for calculating products. How many elements are there of order 3? How many of order 7?
2. a) What is the order of  $\text{Aut}(C_p \times C_p)$   $p$  a prime?  
b) Show that if  $n = 3m$  and  $m^{(>1)}$  is not square-free, then there is a nonabelian group of order  $n$ .
3. What is the commutator subgroup of  $S_4$ ? Prove that  $S_4$  is solvable.
4. Let  $G = D_{10} = \langle a, b : a^2 = b^{10} = 1, aba^{-1} = b^{-1} \rangle$ .  
a) Describe the normalizer of the subgroup  $\langle a \rangle \langle b^5 \rangle$  (i.e.  $N_{D_{10}}(\langle a \rangle \langle b^5 \rangle)$ ).  
b) Describe the commutator subgroup of  $D_{10}$ .
5. Prove that a group of order  $p^n$  ( $p$  a prime), has a subgroup of order  $p^m$  for each  $0 \leq m \leq n$ .